The critical ultraviolet behaviour of $\mathcal{N}=8$ supergravity amplitudes

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We analyze the critical ultraviolet behaviour of the four-graviton amplitude in $\mathcal{N}=8$ supergravity to all order in perturbation. We use the Bern-Carrasco-Johansson diagrammatic expansion for $\mathcal{N}=8$ supergravity multiloop amplitudes, where numerator factors are squares of the Lorentz factor of $\mathcal{N}=4$ super-Yang-Mills amplitudes, and the analysis of the critical ultraviolet behaviour of the multiloop four-gluon amplitudes in the single- and double-trace sectors. We argue this implies that the superficial ultraviolet behaviour of the four-graviton $\mathcal{N}=8$ amplitudes from four-loop order is determined by the factorization the $\partial^8 \mathcal{R}^4$ operator. This leads to a seven-loop logarithmic divergence in the four-graviton amplitude in four dimensions.

INTRODUCTION

There have been recent tremendous progresses in the evaluation of multiloop amplitudes in maximally supersymmetric string theory [1–3], in $\mathcal{N}=8$ supergravity in various dimensions [4–6], and the analysis of the constraints from the extended supersymmetry on possible counterterms to ultraviolet divergences [7–10]. It has been shown that up to and including four-loop order that the four-graviton amplitudes in $\mathcal{N}=8$ supergravity in four dimensions are free of ultraviolet divergences [4–6]. One important question is to determine when the first ultraviolet divergence appears in four dimensions. In this work we argue that from four-loop order the critical ultraviolet behaviour of the four-graviton amplitude is controlled by the factorization of the $\partial^8 \mathcal{R}^4$ operator. This implies that the seven-loop four-graviton amplitude is logarithmically ultraviolet divergent in four dimensions with a counterterm given by the $\partial^8 \mathcal{R}^4$ interaction.

SUPERFICIAL ULTRAVIOLET BEHAVIOUR OF $\mathcal{N}=8$ AMPLITUDES

The mass dimension of the L-loop gravity amplitude in D dimensions is given by

$$[\mathfrak{M}_{n,L}^{(D)}] = \mathrm{mass}^{(D-2)L+2}. \tag{1}$$

In $\mathcal{N}=8$ supergravity (and superstring theory) half of the supersymmetry are explicitly realized at each loop order and any amplitude between any four massless states $\phi_1,\ldots,\phi_4$ in the $\mathcal{N}=8$ supergraviton multiplet take the form

$$\mathfrak{M}_{4,L}^{(D)}(\phi_1,\ldots,\phi_4) = \mathbf{R}^4 \, \mathcal{I}_{4,3}^{(D)}(k^1,\ldots,k^4), \tag{2}$$

where $\mathcal{I}_{4,L}^{(D)}(k^1,\ldots,k^4)$ does not depend on the helicities of the external states, and is of superficial mass dimension $(D-2)L-6$. The operator $\mathbf{R}^4$ is the dimension eight operator defined from the massless four-point $\mathcal{N}=8$ supergravity tree-level amplitude

$$\mathbf{R}^4 = stu\, \mathfrak{M}_{4,tree}^{(D)}(\phi_1,\ldots,\phi_4), \tag{3}$$

where $s = -(k^1+k^2)^2$, $t = -(k^1+k^4)^2$ and $u = -(k^1+k^3)^2$. In the case of the four-graviton amplitudes the tensorial factor in (3) will be denoted $\mathcal{R}^4$. If one parameterises the superficial power counting of the ultraviolet behaviour of the amplitude as

$$[\mathfrak{M}_{4,L}^{(D)}] = \Lambda^{(D-2)L-6-2\beta_L} \, \partial^{2\beta_L} \mathcal{R}^4, \tag{4}$$

where Λ is a momentum cutoff, the critical dimension for ultraviolet divergences in the four-graviton amplitude is given by

$$D \geq 2 + \frac{6 + 2\beta_L}{L}. \tag{5}$$

Up to an including four-loop the supersymmetry constraints [2, 3] implies that $\beta_L = L$.

If only the large-λ regulator of the pure spinor string formalism is used one can show [2, 3] that $\beta_L = L$ for $L \leq 6$ and $\beta_L = 6$ for $L \geq 6$ leading to the critical dimension for the appearance ultraviolet divergence

$$\begin{aligned} D &\geq 4 + \frac{6}{L}; && \text{for } L \leq 6 \\ D &\geq 2 + \frac{18}{L}; && \text{for } L \geq 6. \end{aligned} \tag{6}$$

This rule implies a logarithmic nine-loop divergence in the four-graviton amplitude in four dimensions [3]. From genus five possible divergences from the tip of the pure spinor cone that would require the use of the complicated small-λ regulator [11, 12] can restrict $\beta_L = 4$ for $L \geq 4$ leading to the critical dimension for the appearance of ultraviolet divergence

$$\begin{aligned} D &\geq 4 + \frac{6}{L}; && \text{for } L \leq 4 \\ D &\geq 2 + \frac{14}{L}; && \text{for } L \geq 4. \end{aligned} \tag{7}$$

This rule implies a logarithmic seven-loop divergence in the four-graviton amplitude in four dimensions.

The issue of correctly identifying the ultraviolet behaviour of a supersymmetric theory is equivalent to the understanding of which interactions are true F-terms satisfying non-renormalisation theorems, and which interactions are D-terms receiving quantum corrections to all orders in perturbation [13]. We now discuss the D-terms arising in the four points amplitudes in $\mathcal{N} = 4$ super-Yang-Mills (SYM) in various dimensions before turning to the case of $\mathcal{N} = 8$ supergravity.

PARAMETERIZATION OF THE $\mathcal{N} = 4$ SUPER-YANG-MILLS AMPLITUDES

By partial integration over the superspace variables it is possible to rewrite D-term as "fake" F-term, and

$$
\begin{aligned}
g_{\mathrm{YM}}^{-2L-2} \mathfrak{A}_{4,L}^{(D)} = {} & t_8(F^1 F^2 F^3 F^4)\mathrm{tr}(T^1 T^2 T^3 T^4)\, (k^1 \cdot k^2) \int \prod_{i=1}^{L} \frac{d^D \ell_i}{(2\pi)^D} \frac{t_1(\ell_i, k^i)}{\prod_{j=1}^{3L+1} \ell_j^2} + perm(2,3,4) \\
& + t_8(F^1 F^2 F^3 F^4)\, \mathrm{tr}(T^1 T^2)\mathrm{tr}(T^3 T^4)\, (k^1 \cdot k^2)\, k_m^a k_n^b \int \prod_{i=1}^{L} \frac{d^D \ell_i}{(2\pi)^D} \frac{t_2^{mn|ab}(\ell_i, k^i)}{\prod_{j=1}^{3L+1} \ell_j^2} + perm(2,3,4) \,.
\end{aligned}
\tag{8}
$$

The Lorentz scalar $\mathcal{F}^4 \equiv t_8(F^1 F^2 F^3 F^4)$ is defined by the tree-level four-gluon amplitude

$$
\mathfrak{A}_{4,\mathrm{tree}}^{(D)} = g_{\mathrm{YM}}^2 \frac{\mathcal{F}^4}{stu} \left[u\, \mathrm{tr}(T^1 T^2 T^3 T^4) + perm(2,3,4) \right]. \tag{9}
$$

For large values of the loop momenta ℓ_i the numerator factors in (8) behave as $t_1(\ell_i, k^i) \sim \ell^{2L-4}$ and $t_2^{mn|ab}(\ell_i, k^i) \sim \ell^{2L-6}$. [We have use made explicit the presence of a $s = -2(k^1 \cdot k^2)$ factor in the s-channel diagram [4–6]. The t- and u-channel have the same property after permutation of the labels of the external states.] The $\partial^2 t_8 \mathrm{tr} F^4$ carries the critical ultraviolet behaviour of the four-point amplitudes from two-loop order that have ultraviolet divergences in $D \geq 4 + 6/L$ dimensions. The $\partial^4 t_8 (\mathrm{tr} F^2)^2$ carries the critical ultraviolet behaviour for the double-trace contribution of the four-point amplitude from three-loop order which has ultraviolet divergences in $D \geq 4 + 8/L$ dimensions [1].

A different parameterization of Yang-Mills amplitudes based on cubic interactions have been introduced by Bern, Carrasco and Johansson at tree-level in [14] and at loop orders in [15, 16]. In this parameterization the L-loop four-point amplitudes take the form

$$
\mathfrak{A}_{4,L}^{(D)} = g_{\mathrm{YM}}^{2L+2} \sum_j \int \prod_{i=1}^{L} \frac{d^D \ell_i}{(2\pi)^D} \frac{1}{S_j} \frac{n_j c_j}{\prod_{r=1}^{3L+1} p_r^2}, \tag{10}
$$

detecting the true D-term nature of an interaction can be non-trivial. In the four-point open string amplitudes with $U(N)$ gauge group the true D-term nature of the $\partial^2 t_8 \mathrm{tr} F^4$ and $\partial^4 t_8 (\mathrm{tr} F^2)^2$ interactions became manifest by the appearance of inverse derivative factors arising when integrating over the string theory moduli [1]. It was shown in this analysis that at three-loop order the integration over the open string moduli generates one inverse derivative factor $1/(k^a \cdot k^b)$ in the single trace sector. From four-loop two inverse derivative factors $1/(k^a \cdot k^b) \times 1/(k^c \cdot k^d)$ arise in the single trace and the double-trace sectors. It is important to realize that these inverse derivative contributions do not lead to poles in the total amplitude, they only reduce the number of external momenta factorizing the amplitude.

One can represent the four-gluon amplitudes in $\mathcal{N} = 4$ SYM from four-loop order in a form where the ultraviolet behaviour is explicit in single-trace and double-trace sectors

where the sums now run over all distinct n-point L-loop diagrams with cubic vertices. The inverse propagators of the diagram are p_r^2, c_i are color factors, n_i are Lorentz factors and S_j are symmetry factors of the corresponding Feynman diagrams [15]. The Lorentz factors n_i are constrained by the Jacobi-like relations $n_i + n_j + n_k = 0$ similar to the one satisfied by the color factors $c_i + c_j + c_k = 0$. Such relations, that are necessary for a correct parameterization the amplitudes, where shown to be a consequence of the monodromy relations between open string amplitudes [17] (see as well [18–20]).

Instead of the form of the amplitude given in (8), where the ultraviolet behaviour is explicit, we express the higher-loop SYM amplitudes on a basis of integrals functions containing 1-particle reducible graphs with $1/(k^a \cdot k^b)$ factors cancelling corresponding contributions in the numerators. This way we are mimicking the appearance of the inverse derivative factors noticed in the open string analysis [1].

In this basis a first class of integral functions contributing to the sum in (10), is composed by 1-particle irreducible (1PI) diagrams with Lorentz factor given by

$$
n_j^{1PI} = \mathcal{F}^4 \times (k^1 \cdot k^2)\, k_m^a k_n^b\, t_{j,1PI}^{mn|ab}(\ell_i, k^i), \tag{11}
$$

in the s-channel. For the large values of the loop mo-

menta ℓ_i this factor has the behaviour

$$\lim_{\ell_i \to \infty} t_{j,1PI}^{mn|ab}(\ell_i, k^i) \sim \ell^{2L-6}. \qquad (12)$$

In order to reproduce the correct ultraviolet behaviour of the $\mathcal{N} = 4$ SYM four-point amplitude (8), one needs a second class of integral functions with 1-particle reducible (1PR) graphs with propagator contributions of the type $1/(k^a \cdot k^b)$. As in the open string theory analysis of [1] these reducible graphs do not lead to massless pole in the total amplitude because the $1/(k \cdot k^b)$ factor are cancelled by corresponding factors in the numerator.

At three-loop order one only needs 1-particle reducible integral functions with one factor of inverse derivative [1, 15] with numerators given by the Lorentz factor

$$n_j^{1PR}|_{L=3} = \mathcal{F}^4 \times (k^1 \cdot k^2)^2 \, t_j^{1PR,L=3}(\ell_i, k^i), \qquad (13)$$

in the s-channel. Examples of such three-loop contributions are given by the diagrams (j)–(l) of fig. 2 in [15]. For large values of the loop momenta ℓ_i this factor has the behaviour

$$\lim_{\ell_i \to \infty} t_j^{1PR,L=3}(\ell_i, k^i) \sim 1. \qquad (14)$$

From four-loop order one needs 1-particle reducible integral functions with two inverse derivative factors [1]. Such integral functions have numerator Lorentz factors given by

$$n_j^{1PR} = \mathcal{F}^4 \times (k^1 \cdot k^2)^3 \, t_j^{1PR}(\ell_i, k^i), \qquad (15)$$

in the s-channel. For large values of the loop momenta ℓ_i this factor has the behaviour

$$\lim_{\ell_i \to \infty} t_j^{1PR}(\ell_i, k^i) \sim \ell^{2L-8}. \qquad (16)$$

This parameterization of the four-point multiloop $\mathcal{N} = 4$ SYM amplitude reproduces the critical ultraviolet behaviour [1].

PARAMETERIZATION OF THE $\mathcal{N} = 8$ SUPERGRAVITY AMPLITUDES

The question of D-term and F-term interactions in $\mathcal{N} = 8$ supergravity can be approached in a similar fashion as in the SYM case discussed in the previous section. The pure spinor formalism indicates [2] that the $\mathcal{R}^4$, $\partial^4 \mathcal{R}^4$ and $\partial^6 \mathcal{R}^4$ interactions are F-terms represented by integrals over the pure spinor superspace involving only the regulator for the large-value of the pure spinor ghost [2]. These interactions receive only a finite number of loop corrections in string perturbation [3, 21]. There are indications from string/M-theory computations that $\partial^8 \mathcal{R}^4$ term is a D-terms [21]. The analysis using the pure spinor formalism indicates that from five-loop order the

amplitudes receive contributions from the small-λ regulator and inverse derivative factors from the integration over the string moduli [1].

With the parameterization (10) of the Yang-Mills amplitudes it was proposed in [15] that the multiloop four-point supergravity amplitudes take the symmetric form

$$\mathfrak{M}_{4,L}^{(D)} = \kappa_{(D)}^{2L+2} \sum_j \int \prod_{i=1}^{L} \frac{d^D \ell_i}{(2\pi)^D} \frac{1}{S_j} \frac{n_j \tilde{n}_j}{\prod_{r=1}^{3L+1} p_r^2}. \qquad (17)$$

This parameterization of the supergravity multiloop amplitudes is particularly well suited for incorporating the inverse derivative effects. The left-moving n_j and right-moving $\tilde{n}_j$ Lorentz factors have a generalized gauge freedom leaving the amplitudes invariant [15, 16]. We make the choice that $\tilde{n}_j$ is n_j expressed in terms of the right moving polarisations.

For the 1-particle irreducible integral functions contribution to (17) the numerators are given by the square of the SYM Lorentz factors in (11)

$$n_i^{1PI} \tilde{n}_i^{1PI} \sim k^8 \, \mathcal{R}^4 \times (t_{j,1PI}^{mn|ab}(\ell_i, k^i))^2. \qquad (18)$$

Where we used that $\mathcal{R}^4 = \mathcal{F}^4 \tilde{\mathcal{F}}^4$ which is a direct consequence of the KLT relation [22] between the four-point tree-level amplitude in gravity (3) and in Yang-Mills (9). Using the behaviour (12) for the large values of the loop momenta $\ell_i \sim \Lambda \gg 1$, the superficial ultraviolet behaviour of the 1-particle irreducible integral function in (17) is $\partial^8 \mathcal{R}^4 \Lambda^{(D-2)L-14}$.

The three-loop supergravity amplitude (17) contains the 1-particle reducible graphs bringing $1/(k^a \cdot k^b)$ contributions [1, 15]. The numerators are given by the square of the SYM Lorentz factors in (13)

$$n_j^{1PR}|_{L=3} \tilde{n}_j^{1PR}|_{L=3} \sim k^8 \, \mathcal{R}^4 \times (t_j^{1PR,L=3}(\ell_i, k^i))^2. \qquad (19)$$

Because these 1-particle reducible graphs have a single $1/(k^a \cdot k^b)$ factor and using the behaviour (14) for the large values of the loop momenta $\ell_i \sim \Lambda \gg 1$, the superficial ultraviolet behaviour of these diagrams is $\partial^6 \mathcal{R}^4 \Lambda^{3(D-6)}$. This is the correct leading ultraviolet behaviour, given by (4) with $\beta_3 = 3$, for the three-loop four-graviton amplitude [3, 5]. For $L = 3$ the contribution from the 1-particle irreducible graphs in (18) give the milder ultraviolet behaviour $\partial^8 \mathcal{R}^4 \Lambda^{3D-20}$ of the diagrams (a)–(i) in fig. 2 of [15] (see as well [5]).

From four-loop order the contributions to the 1-particle reducible integral functions to (17) have two inverse derivative factor [1] with numerators given by the square of the SYM Lorentz factor in (15)

$$n_j^{1PR} \tilde{n}_j^{1PR} \sim k^{12} \, \mathcal{R}^4 \times (t_j^{1PR}(\ell_i, k^i))^2. \qquad (20)$$

Because these 1-particle reducible graphs have double inverse derivative factors, $1/(k^a \cdot k^b) \times 1/(k^c \cdot k^d)$, and using

the behaviour (16) for the large values of the loop momenta $\ell_i \sim \Lambda \gg 1$, the superficial ultraviolet behaviour of the supergravity amplitudes (17) at $L \geq 4$ loops is $\partial^8 \mathcal{R}^4 \Lambda^{(D-2)L-14}$. This correspond to the critical ultraviolet behaviour with $\beta_L = 4$ for $L \geq 4$ given in (7).

Since the $\mathcal{N} = 8$ supergravity amplitudes in (17) are composed by integrals with numerators given by squares summed over with positive relative coefficients given the symmetry factors of the diagrams, it is unlikely that cancellations leading to a better ultraviolet behaviour can arise without modifying the ultraviolet behaviour of the $\mathcal{N} = 4$ SYM amplitudes. We therefore conclude that the superficial ultraviolet behaviour of the four-graviton $\mathcal{N} = 8$ supergravity amplitudes is given by the rule in (7). This implies that the four-graviton amplitude has a seven-loop logarithmic divergence in four dimensions with the $\partial^8 \mathcal{R}^4$ interaction for counterterm. A candidate on-shell superspace E_7-invariant expression for this seven-loop counterterm is the volume of superspace

$$\int d^4x \sqrt{-g^{(4)}} \, \partial^8 \mathcal{R}^4 \sim \int d^4x \int d^{32}\theta \, |E| \,, \qquad (21)$$

where $|E|$ is the determinant of the super-vierbein. To the contrary to the volumes of $\mathcal{N} = 1$ and $\mathcal{N} = 2$ superspace this expression does not seem to be vanishing [10].

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